AN APPLICATION OF A HEURISTIC APPROACH IN A 6^{th} GRADE MATHS LESSON

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Abstract. *The article presents a model of a lesson on the topic: "Direct proportion Graphs" intended for the 6^{th} grade students. A methodical system of problems is also included in the paper*

Key words: heuristic approach, direct proportion, graphs.

Nowadays, one of the most important tasks of the modern school is the organization of the educational school in a way that provides conditions and opportunities for the formation and development of students' mathematical abilities and creative potential. Achieving this goal depends on teachers' ability to manage effectively students' heuristic activities.

The teacher's role in the process of learning is undoubtedly the leading one. However, the teacher's role in the conventional education, is just to provide knowledge, which should be acquired by the students, while in the heuristics education this role turns into an even more important one because it is supposed that in this way the teacher will help students to set their own learning goals, to create an action plan and find their way in reaching the discovery.

The idea of the person-oriented approach in the field of education dates back to the early 20th century (Frene, Montessori, Dewey, etc.). Individualization and differentiation in education are basic characteristics of the ideas of the modern educational technologies too. Moreover, taking into account the orientation towards the personality, the technology process of education should be based on the creative approach in the organization of the training [6].

The key components of the heuristic education are: motivation, problematization of the activity, solution of the problem by the participants; demonstration of educational products and their comparison; reflection on results. The heuristic educational situation (according to A. Khutorskoy; E. Skafa and V. Milushev) is in the center of the heuristic learning. The problematics is a feature of any type of training, but in heuristic training the transition is made from "knowledge to ignorance, from problem to solution, from question to answer", and the main goal of training is to reach a new, previously unknown result of the activity [2].

Having in mind that a concrete "scenario" is not supposed to be a part of the heuristic process, we are going to outline the topic: "Direct Proportion Graphs", using a heuristic approach. The topic is included in the compulsory mathematics curriculum for Year 6. According to the educational goals set out in the methodological instructions for the realization of the lesson, the students must acquire knowledge and skills for recognizing and constructing the graph of direct proportion. The topic is considered to be a propaedeutics of the concept of function. Intended to be suitable for 6-th grade students, only a special case of the linear function, $y = k.x$ is considered to be included.

Using the mathematical problems that we will consider, we implicitly introduce the idea of finding an equation of a straight line. Thus, using a heuristic approach, some of the material or mathematical problems intended for study in higher grades and/or in the elective courses could be introduced to the students much earlier, which is not the main subject of this article.

First didactic task of the lesson includes checking homework, then activating prior knowledge and motivation. At this stage, students must be involved in the heuristic process. If it is possible, a heuristic dialogue could be used for checking homework. To activate knowledge teacher may use specific questions and math problems which are often based on the reproductive or partial-search level of cognitive activity.

After that, the following math problems could be used for activating knowledge:

Problem 1. Which of the following is in direct proportion?

A) Speed (V) and time (t) in which a given distance (S) is travelled;

B) The distance travelled (S) and Speed (V) in a fixed time interval;

C) The actual distance between two towns and the distance between them on a map at a given scale;

D) The capacity of vessels and the number of the latter needed to store a certain amount of water;

E) The number of the pieces made and the number of workers for a fixed time interval.

Problem 2. Students are asked to come up with their own examples which are discussed by the class in order to raise new, hitherto unclear issues.

Moving ahead with the topic of the lesson and applying the concept of direct proportionality in examples of real-life situations which students are familiar with, we proceed with Problem 3. It was given as an optional homework. Students had to record the distance they had travelled and the time interval they had covered this distance as they moved with the same speed between two points. The context of the task was not chosen at random. According to certain researches (reviewed in [1]) students deal better with graphs when one of the variables is "time" or "time dependent".

As Problem 3 is not mandatory, the teacher must be able to present a graphic illustration of a hypothetical situation for this case. The following (Figure 1) information is presented on the board:

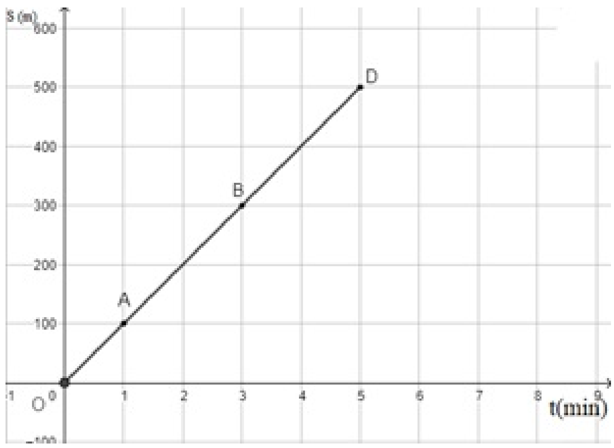

Figure 1.

The aim is to provoke a dialogue, mentally to "walk" the way from the known to the unknown, to find issues that have not been considered so far. At this stage of the lesson, the following questions are relevant: "What is given?", "How will you interpret the information presented?".

It should be noted that the topic "Interpretation of data presented by diagrams" is included in the the curriculum of fifth grade students. Therefore the given situation is familiar to the students. Answering the questions such as: "How many minutes did it take me to get to point A?";

What is the distance between A and B?", etc. we move on to the answer of the question: "What is the speed of movement?" Once again, students find the relationship between two variables direct relationship between the distance (S) and time (t). And thus, implicitly, we move smoothly to the idea to present graphically the relation between these variables. Of course, some of the students ask: "Is it possible to present it that way?"; "Is this particular example the only one that could be referred to the considered graph?". However, there is still a possibility that not all of the students have already seen a new problem. The teacher should understand what is going on in his student's mind. That's why, intending to provoke a problematic point for students, the following problem 4 is presented to them. Then we gradually move to the next stage of the lesson.

x	y
-2	-4
-1	-2
0	0
1	2
2	4
3	6

Table 1.

Problem 4. The following problem is based on the idea of searching for an algorithm. The teacher creates Table 1.

Here are some applicable questions the teacher may have in mind: "What data do we have?"; "Is there a relationship between the values of the variables recorded in the left and the right columns?"; "Can you find a rule by which we can get the values of y from the values of x?". It should be noted that the search for algorithms can be seen as an essential part of the process of developing students' creative thinking. Searching for algorithms actually is a heuristic activity. Students discover an algorithm by themselves and they can use it in other similar math problems [5].

When considering this problem and its problematization, the students are being provoked to make attempts, to look for different ways in which the values of one of the variables can be obtained from the values of the other variable and finally the students discover the rule by themselves. The discussion can be in different directions, as well as the dialogue. The students come to the conclusion: the values of x are doubled and the corresponding values of y are obtained. Finding the rule, they easily write the formula: $y = 2.x$.

At this stage we use generalization – we keep the common between all recorded equations, as well as abstraction – we omit the difference between the equations – i.e. the specific numerical values of x and y. The heuristic guidance that can be used is: "Summarize your reasonings".

Another option that students may have in mind is to record the data as coordinates of points. The idea of presenting as points on the rectangular coordinate system mostly requires more preparation, as it is related to prior knowledge. However, considering the solution of Problem 3, the students may come up with this idea. When the students conceive the above-mentioned idea and the teacher encourage them to realize it, the graphical representation of direct proportionality follows. Then we draw their attention to the points that are plotted on the rectangular coordinate system – a finite number of points, on the one hand, but on the other – they already know that for each value of x, they will get a value for y. Often the questions that need to be addressed to the students are suggested by them. For example: (1) "Is this particular segment the only one that represents the graph of direct proportionality?"; (2) "Does the graph always pass through the origin of the coordinate plane?"; (3) "Considering the example, the graph is located in the first and third quadrants of the coordinate plane. Is it always the same?" Based on the students' questions (or those provoked by the teacher), the students raise hypotheses and "prove" them as far as their knowledge at the time allows.

Looking for answers to these questions ((1) in particular), the students can easily convince themselves that by choosing an arbitrary value of x, they will get a value of y, and the point determined by these coordinates will lie on the same graph. At this stage, as a result of inductive reasoning, the hypothesis that all points of the graph lie on one line is accepted. In regard to the question (2) – the students could be advised to choose another value for k, ($k \neq 0$). After several attempts and on the basis of specific examples, students come to the conclusion that for each k, $k \neq 0$, the point O lies on the graph of direct proportionality. They also have to prove it. For each k, $k \neq 0$; $0 = k.0$ is a true numerical equality.

When drawing the graph it is necessary to pay attention to the fact that if the point O lies on the graph, it is enough to find the coordinates of just another point that belongs to this line. The students can also come to this conclusion on their own, provoked by teachers' questions.

The examples we are looking at to find the answers to question (1) can also be used for question (3). For example, when we have $y = -3x$; $y = -x$; $y = -\frac{1}{3}$, the graphs are located in the second and forth quadrant. If $y = \frac{1}{3}x$; $y = x$ and $y = 3x$, the graphs are in first and third quadrant. Thus, using a concrete inductive approach, the examples of different values of the coefficients are considered and it is concluded that for $k > 0$ the graph is located in the first and third quadrant, and for $k < 0$ the graph is located in the second and fourth quadrant. When considering each of the questions, the teacher has an appropriate guiding role. The questions that the teacher asks can turn to the logical consequence and provoke the students to discover things by themselves. After we have taken the maximum of the issues under consideration and based on the previous examples, we move on to the methodological system of problems presented below. Based on the experience gained, we believe that the solving problems for plotting direct proportionality (which requires an algorithmic approach) is sufficiently practiced in the examples discussed above (3). We start with problems that suggest the application of the theoretical issues:

Problem 5.1. In which quadrants is the graph of the direct proportionality located:

$$\text{a) } y = 1,25x; \quad \text{b) } y = -\tfrac{17}{29}x; \quad \text{c) } y = 568x?$$

Problem 5.2. Do the points belong to the graph of direct proportionality $y = -\frac{1}{2}x$

$$\text{a) } (-4; 2); \quad \text{b) } (2; -1); \quad \text{c) } (2020,2; -1010,1).$$

The solution of the problem can begin with the questions: “What are the data?”; “What is the condition?”; “Could you restate the problem?”; “What do you expect to find?”At the next stage of solving the problem, once we are convinced that the students have understood the problem, it is good to teach them to ask themselves and to answer the questions: Have you seen it before?”. If the answer is positive: “Can you use the same method, used to solve the previous problem? Some of the students would appropriately suggest the idea to draw the graph of the direct proportion and to check if the points belong to this graph. The teacher needs to help them convince themselves that this idea is not always applicable, giving an example as follows for c) $(2020; -1010)$.

Solution to the last problem aims to show the need to look for another way to solve the problem, analytically, we replace with the corresponding values of x and check whether a true numerical equality is obtained. We believe that it is important for students to understand the reasoning behind the solutions to the problems. What questions they need to ask themselves, to get used to such reasonings, to become aware of the way a task has been solved.

Problem 5.3. Find k if the graph of direct proportionality $y = k.x$ passes through the points:

$$\text{a) } A(3;6); \quad \text{b) } B(-5;-15); \quad \text{c) } C(9;-3).$$

The solution of this problem requires a direct application of the above – to replace in $y = k.x$ with the corresponding values of x and y and to solve an equation, from which the corresponding value of k is found.

Proceeding from problems 5.2 and 5.3, the following problem 5.4 is proposed. component part of problem 5.4 is the solution of the two previously considered problems.

Problem 5.4. [4].

a) The graph of direct proportion $y = k.x$ passes through the point $(-1;-2)$. Find which points belong to the graph

$$\text{A) } (-\tfrac{1}{2};-1); \quad \text{B) } (0,2;0,4); \quad \text{C) } (10;20); \quad \text{D) } (30;-15).$$

b) The graph of direct proportion $y = -k.x$ passes through the point $(-4;2)$. Find which points belong to the graph:

$$\text{A) } (-2;1); \quad \text{B) } (2;-1); \quad \text{C) } (-\tfrac{1}{2};-\tfrac{1}{4}); \quad \text{D) } (-50;-25).$$

Problem 5.5. Direct proportion $y = k.x$ is given.

a) Find the coefficient of proportionality and fill in the Table 2.

b) Draw the graph of the given relation.

x	-2	-1	0	1	2	3	4
y							16

Table 2.

Next math problem represents a real-life situation. It has a practical focus. It is necessary to transfer the knowledge acquired so far in another context, to reflect their practical applicability.

Problem 5.6. Kalina makes 2 bracelets in 5 minutes.

a) Find a formula for the relation (number of bracelets) which are made in x minutes ($x > 0$).

b) Draw the graph of this relation.

The last problem suggests applying the ideas and knowledge discussed above and those from the topic "Pythagorean theorem":

Problem 5.7. [5] In a rectangular coordinate system, plot the points $B(-1;-2)$ and $D(2;2)$. Plot point C – symmetric to point D with respect to the abscissa and point E – symmetric to D with respect to the ordinate.

a) Draw the line OD and write the coefficient k of the direct proportionality, whose graph is this line. Is it true that the points C and E belong to the graph of direct proportionality $y = -k.x$?

b) Is there a direct proportionality whose graph is the line BD? Justify it.

c) How many units is the segment BD equal to?

Answer:

a) $y = x$, $k = 1$;

b) No, because it does not pass through the origin of the coordinate system;

c) 5.

When solving each of the problems it is useful to apply the model of Polya: "1) Understanding the problem; 2) Devising a plan; 3) Carrying out the plan 4) Looking back" [3].

At the next stage of the lesson, the students could be provided with the opportunity to design, observe, invent examples that reflect direct proportionality and as well as to present this relationship graphically. In the last part of the lesson students are asked to summarize the main points of the lesson. Each of them brings out the information essential for him/her and a reflective activity is performed as far as it is possible. It should be noted that using a heuristic approach in teaching and to problem-solving, reflective activity is carried out after each example and each problem solved.

In conclusion, teaching mathematics on the basis of the heuristic approach often requires more time than the traditional approaches. But it's not wasted time, because new knowledge acquired and problems solved are result of students own effort and experience of reasoning.

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